Theory and Methodology

On the optimization of supply chain networking decisions

Salem Lakhal a, Alain Martel a,*, Ossama Kettani a, Muhittin Oral b

a Network Organization Technology Research Center, Faculté des Sciences de l’Administration, Université Laval, Cité Universitaire, Ste-Foy, QC, Canada G1K 7P4
b Graduate School of Management, Sabanci University, Istanbul, Turkey

Received 6 March 2000; accepted 14 May 2000

Abstract

Companies strive to position themselves to maximize the value they add to the supply chains in which they are embedded. This raises strategic questions such as: Which durable resources should be developed to enhance current core competencies? Which activities should be externalized and to which potential partner should they be given? Which internal activities should be preserved and developed? How should the resources of the enterprise be allocated to activities? The aim of this paper is to propose a mathematical programming model of the extended enterprise which can be used to investigate this type of strategic networking issues. A number of general network modeling constructs are first proposed. A model to optimize the supply chain structure under specific assumptions on the nature of production, cost and value functions in typical production/distribution companies is then derived. A heuristic to obtain solutions from the model is also presented. Finally, an example based on a refrigerator company is used to illustrate the usefulness of the approach. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Supply chain strategy; Networking; Extended enterprise; Theory of the firm; Mixed integer programming

1. Introduction

Manufacturing and distribution companies now tend to see themselves as part of larger supply chains forming a network of geographically dispersed, but functionally integrated, activities. The position a company occupies in the various supply chains in which it is involved is a major strategic issue. To develop a sustainable competitive advantage, companies concentrate on their core competencies (Prahalad and Hamel, 1990) and, when they find that a non-strategic activity can be performed more effectively by a third-party, they tend to externalize it. The core competencies a company possesses depend heavily on its resources and on how they are used. When a company is able to develop and allocate resources to activities in a way which creates more value for customers than competitors can, it creates a sustainable competitive advantage. A superior supply chain strategy is then one which maximizes the value
added by internal activities while developing solid partnerships leading to high value external activities.

This vision of supply chain strategy stems from the activity-based (Porter, 1985, 1991) and the resource-based (Peteraf, 1993; Montgomery, 1997) views of the firm developed in the literature in recent years. According to Porter, the sources of competitive advantage of an enterprise mainly center around its activities: its activities that determine relative cost, differentiation attributes, and hence customer value, from which potential profit ultimately derives. In this context, a firm’s strategy is manifested in the way in which it configures and links the many activities in its value chain relative to competitors. On the other hand, the resource-based view of the firm asserts that superior companies possess heterogeneous resources that differentiate them from other enterprises and allow them to earn sustainable profits, because there are forces which limit competition for critical resources. Lakhal et al. (1999) use a graph theoretic model of network companies to show that these two views are in fact duals of one another. This literature provides conceptual frameworks which are helpful for strategy formulation, but it does not suggest any formal decision support tools to guide strategic networking decisions. Shapiro (1999) takes an initial step in that direction by examining the relationships between activity based costing (ABC), optimization models for strategic decision support, and the resource-based view of the firm. He discusses several modeling issues and proposes an approach to extend ABC relationships to an optimization model. Starting from the analytical framework of Lakhal et al. (1999), the aim of this paper is to propose a mathematical programming model of the extended enterprise which can be used to support strategic networking decisions.

More specifically, the problem examined is the following. Given

- an extended enterprise and its supply chain partners, as well as the activities currently performed by the resources of the enterprise and by its partners,
- a set of networking opportunities, a number of resource acquisition or disposal possibilities, as well as some activity reengineering alternatives,
- the economies of scale associated to various outsourcing and resource utilization strategies as well as the impact on market share of various pricing strategies,
which extended enterprise structure would maximize the value added (profit) by internal activities? In other words, the model proposed should help the enterprise resolve the following strategic questions:

- Which resources should be preserved and developed to enhance current core competencies?
- Which activities should be externalized and to which potential partner should they be given?
- Which potential internal activities should be preserved and developed?
- How should the resources of the enterprise be allocated to its activities?
- What level of activity should be targeted for each internal unit and for each external partner?

The paper is organized as follows. The stage is set by introducing some basic activity, resource and value modeling constructs and by presenting a generic model of the extended enterprise. Some specific assumptions are made about the nature of production, cost and value functions in typical production/distribution companies, and a mathematical programming model is derived to optimise the supply chain structure under these conditions. A heuristic to obtain a solution from the model is also presented. Finally, the usefulness of the approach is illustrated with an example.

2. Generic activity networking model

The basic components used to model the extended enterprise are activities, resources and products. An activity uses durable resources to transform input products into output products. The set of activities of an extended enterprise (firm) $f$ is denoted $A_f$. It can be partitioned into the set $A_i^f$ of its internal activities and the set $A_e^f$ of the external activities performed by business partners. $A_i^f$ is the union of external input (source) and external output (client) activity sets $A_i^o$ and $A_e^o$. The activities considered in the model include all the enterprise current activities, plus any new
internal or external (potential partners) activities that could be introduced to improve the performance of the company.

The set of all distinct durable resources of a firm \( f \) is denoted by \( R_f \). The amount, under some appropriate metric, of durable resource \( r \) available during the planning horizon considered is denoted by \( x_r \), and the column vector of these amounts by \( \mathbf{x} \). The set of the durable resources associated with an activity \( a \) is denoted by \( R_a \subset R_f \). The amount of durable resource \( r \) used by an internal activity \( a \) during the time horizon considered is denoted by \( x_{ar} \), the vector of resources used by activity \( a \) by \( \mathbf{x}_a \) and the matrix of all the \( x_{ar} \)'s by \( \mathbf{X} \). The set of resources considered includes all the resources used by the company at the beginning of the planning horizon, plus any new durable resources acquired to improve the performance of the company.

In a supply chain, an activity uses the products of the immediately preceding activities as inputs, and provides products for the immediately succeeding activities. The last products on the supply chain are the products delivered to market, whereas the first ones are mostly raw materials and basic components. The index used for products is \( p \) and the set of products of the extended enterprise \( f \) is denoted by \( P_f \). For a given activity, an input product can come from several internal or external sources, and an output product can be provided to several destinations. To track this properly, four sets are associated with activity \( a \) in \( A_f^1 \):

- \( P_a^i \) the set of input products,
- \( A_a^o \) the set of source activities,
- \( P_a^o \) the set of output products,
- \( A_a^o \) the set of client activities.

For the horizon considered, the quantity of input product \( p \in P_a^i \) provided to activity \( a \) by predecessor \( i \in A_a^i \) is denoted by \( y_{iap} \), and the quantity of the output product \( p \in P_a^o \) provided by activity \( a \) to successor \( j \in A_a^o \) by \( y_{ajp} \). The set of all products associated with activity \( a \) is \( P_a = P_a^i \cup P_a^o \) and, clearly, \( P_a \subset P_f \). For a given firm \( f \), the set of all products sourced from external activities is \( P_f^e \), the set of all products sent to external activities is \( P_f^e \) and the set of all products coming from and going to internal activities is \( P_f^i \).

An internal activity \( a \) is thus characterized by its resources and by its products, as well as by a method \( m_a \), which specifies how the resources proceed to transform input products into outputs. This method is modeled by a vector valued function

\[
\mathbf{f}_m(a, \mathbf{x}_a, \mathbf{y}_a^o) = \mathbf{0}, \quad a \in A_f^1,
\]

which relates the vector \( \mathbf{y}_a^o \) of the output quantities to the vectors \( \mathbf{y}_a^i \) and \( \mathbf{x}_a \) of the input quantities and durable resource quantities. For external activities, the resources and methods used are not modeled explicitly. Instead, it is assumed that the output vector \( \mathbf{y}_a^o \) for an external source \( a \in A_f^1 \) is restricted to a predetermined domain \( \mathcal{Y}_a^o \) which characterizes its capacity, and that the input vector \( \mathbf{y}_a^i \) for an external destination \( a \in A_f^1 \) is restricted to a predetermined domain \( \mathcal{Y}_a^i \) which characterizes its demand.

Using these constructs, the extended enterprise \( f \) can be modeled as a directed multigraph (network) of activities \((A_f, S_f)\), where \( S_f \) is the set of directed arcs indicating the sequence of the activities required to produce and market a given consumer product in a specific market. An arc \((i, j, p) \in S_f\) corresponds to a product \( p \) being passed from activity \( i \) to \( j \), and it is associated to the flow variable \( y_{ijp} \). The set of input arcs for activity \( a \) is denoted by \( S_a^i \) and the set of its output arcs by \( S_a^o \). For the extended enterprise \( f \), the sets of the internal arcs, of the arcs with an external source and of the arcs with an external destination are denoted by \( S_f^i, S_f^o \) and \( S_f^e \), respectively.

In order to profit from collaboration decisions, an enterprise must be able to alter its durable resource base. It is therefore assumed that the resources available at the beginning of the planning horizon, \( \mathbf{x}_0 \), can be adjusted within certain limits, according to a metric reflecting the nature of each resource (hiring or laying off employees, buying or selling equipment or facilities, etc.). Decisions must be taken as to the amount of resources \( \mathbf{x}^+ \in \mathcal{X}^+ \) to be added or \( \mathbf{x}^- \in \mathcal{X}^- \) to be disposed off, where \( \mathcal{X}^+ \) and \( \mathcal{X}^- \) are the sets of feasible resource addition and subtraction, respectively. For a given resource \( r \in R_f^i \), the amount of resource available during the planning horizon is therefore
Clearly, the decision variables \( x_r \) are discrete or continuous, bounded or unbounded. Not required, the following resource utilization possible that all the resources of the enterprise will not be required, the following resource utilization constraints must be respected for a given resource \( r \in R_f^I \):

\[
\sum_{a \in A_f} x_{a0} + x_r^- - x_r^+ \leq x_r^0, \quad r \in R_f^I.
\]

Resources are not free. The cost of using or owning \( x_r \) units of resource \( r \) during the time horizon considered can take various forms represented here by the cost function \( c_r(x_r) \), the vector of these functions being denoted by \( c(x^+, x^-) \). It is assumed that the price \( u_a \) paid by activity \( a \) for the use of these resources is based on a cost sharing mechanism described by the implicit composite vector valued function

\[
C_f(c(x^+, x^-), X, u) = 0,
\]

where \( u \) is the vector of the price paid by the activities. The nature of this function is discussed in more detail in the next section.

The product \( p \) associated to any arc \((i, j, p)\) of the extended enterprise activity network has a certain value. The amount paid for product \( p \) depends on the extent to which producer-activity \( i \) is able to deliver the attributes client-activity \( j \) wants, as well as on the quantity \( y_{ijp} \) involved. The resulting revenues are modeled by value functions \( v_{ijp}(y_{ijp}) \) and hence the average unit price paid for the product associated with arc \((i, j, p)\) is \( v_{ijp}(y_{ijp})/y_{ijp} \). The function \( v_{ijp}(y_{ijp}) \) can take several forms depending on the nature of the source and the destination. If an arc starts with an external supply source or ends with an external demand destination, for example, its value function may increase up to a certain threshold after which it starts declining. The quantities on arcs involving external destination points may also be limited by market conditions. The determination of the value of the products associated to internal arcs is somewhat arbitrary. Fortunately, as will be shown in the model described, internal value functions are not required.

The objective is to find a networking strategy maximizing the value added by the internal activities of the enterprise, which is equivalent to maximizing profits. Several investigations indicate that this is the objective pursued by most firms (Hornby, 1995). The value added, \( w_a \), by an internal activity \( a \) is obtained by subtracting the costs of the input products and of the resources used from the revenue generated by output products, which is given by the relation

\[
w_a = \sum_{(a,j,p) \in \mathcal{S}_f^o} v_{ajp}(y_{ajp}) - \sum_{(i,a,p) \in \mathcal{S}_f^i} v_{iap}(y_{iap}) - u_a.
\]

The value added by enterprise \( f \), is obtained simply by adding the value added by all its internal activities, which yields

\[
w_f = \sum_{a \in A_f^I} w_a
\]

\[
= \sum_{(a,j,p) \in \mathcal{S}_f^o} v_{ajp}(y_{ajp}) - \sum_{(i,a,p) \in \mathcal{S}_f^i} v_{iap}(y_{iap}) - \sum_{a \in A_f^I} u_a.
\]

Note that since internal arcs are both the input of an activity and the output of an other activity, their value functions cancel when the value added by the enterprise is computed. It is therefore not necessary to use these functions to compute the enterprise profits. This avoids the difficult problem of computing internal transfer prices.

Taking all this into account, it can be seen that to maximize value added, the extended enterprise must solve a mathematical program having the following form:

\[
\text{max } w_f = \sum_{(a,j,p) \in \mathcal{S}_f^o} v_{ajp}(y_{ajp}) - \sum_{(i,a,p) \in \mathcal{S}_f^i} v_{iap}(y_{iap}) - \sum_{a \in A_f^I} u_a \tag{GP}
\]

subject to

- \( \mathbf{f}_m(y_a^i, x_a, y_a^0) = 0 \quad \forall a \in A_f^I \),
- \( y_a^i \in \mathcal{Y}_a^i \quad \forall a \in A_f^I \), \( y_a^0 \in \mathcal{Y}_a^0 \quad \forall a \in A_f^I \),
- \( y_{ijp} \geq 0 \quad \forall (i, j, p) \in \mathcal{S}_f^I \),
• \( \sum_{\alpha \in \Delta^r_j} x_{\alpha} + x_{\alpha}^r - x_{\alpha}^p \leq x_{\alpha}^o \quad \forall r \in R^1_j, \)
  \( x_{\alpha}^r \in \mathcal{X}^r_\alpha, \quad x_{\alpha}^p \in \mathcal{X}^p_\alpha \quad \forall r \in R^1_j, \)
• \( C_f(c(x^-, x^+), X, u) = 0, \quad u \geq 0, \quad X \geq 0. \)

When the firm \( f \) is considered as a single activity, this value-added-maximization model reduces to the classical mathematical programming model of a multi-product, multi-resource firm (Naylor and Vernon, 1969). Program GP can thus be seen as a generalization of the classical model of the firm to the case of a multi-activity extended enterprise.

3. Typical production/distribution systems

In this section, the formulation of production, cost and product value functions will be discussed and a networking model proposed for a typical production/distribution company.

3.1. Production functions and flow variable domains

Most production/distribution companies nowadays use enterprise resource planning (ERP) systems to support their production and distribution activities. These software systems enable the enterprise to effectively maintain the bill of materials and the bill of resources which characterize their material and resource requirements. Bills of materials/resources are inherently linear so that, in this context, the production function \( f_{ma}(y_{ajp}^{i}, x_{a}, y_{a}^{o}) = 0 \) of an activity \( a \in A^j_f \) reduces to the following system of linear equations:

\[
E^a_{ma} y_{ajp}^{i} - B_{a} E^o_{a} y_{a}^{o} = 0, \quad a \in A^j_f,
\]
\[
x_{a} - G_{a} E^o_{a} y_{a}^{o} = 0, \quad a \in A^j_f,
\]

where \( B_{a} \) is a matrix of elements \( b_{ap} \) defined as the quantity of input product \( p \) used by activity \( a \) to produce one unit of output product \( q \), and \( G_{a} \) is a matrix of elements \( g_{ajp} \) defined as the quantity of durable resource \( r \) activity \( a \) requires per unit of output product \( p \), and where \( E^a_{ma}, E^o_{a} \) are 0–1 matrices specifying, for activity \( a \), the arcs \((i,a,q)\) providing input product \( p \) and the arcs \((a,j,q)\) associated to output product \( p \), respectively. More specifically, \( E^a_{ma} \) is a \(|P^a| \times |S^o_{a}| \) matrix and its element \( e_{p,(i,a,q)} \) is equal to 1 if \( q = p \), and 0 otherwise. Similarly, \( E^o_{a} \) is a \(|P^o| \times |S^o_{a}| \) matrix and its element \( e_{p,(a,j,q)} \) is equal to 1 if \( q = p \), and 0 otherwise. The linear system (1) drives the behavior of the flow variables for internal activities.

To reflect the capacity restrictions of sourcing partners, in most cases it is sufficient to impose upper limits on the quantities that can be procured. The domains \( \mathcal{Y}^o_{a}, a \in A^0_f \) can thus be modeled by the bounds

\[
0 \leq y_{ajp} \leq y_{ajp}^{\max}, \quad (a,j,p) \in S^j_f, \quad (2)
\]

where \( y_{ajp}^{\max} \) is the maximum quantity of product \( p \) external activity \( a \) can provide to internal activity \( j \) during the planning horizon considered.

Customer demand, on the other hand, is more naturally expressed in terms of end products requirements by market area, and this independently of the activity responsible for delivery. For this reason, as suggested by Cohen et al. (1989), outbound flows (enterprise to market) are constrained by lower and upper volume limits. The domains \( \mathcal{Y}^i_{a}, a \in A^i_f \) can thus be modeled by the constraints

\[
d_{ma} \leq \sum_{(i,a,p) \in S^i_m} y_{ajp} \leq \bar{d}_{ma}, \quad a \in A^i_f, \quad p \in P^i_m, \quad (3)
\]

where \( d_{ma} \) is a minimal market penetration target fixed by the enterprise and \( \bar{d}_{ma} \) is the potential demand for product \( p \) in market \( a \).

3.2. Resource cost functions and domains

The implicit function \( C_f(c(x^-, x^+), X, u) = 0 \) has been introduced into the model to reflect the durable resources cost sharing mechanism used by enterprise \( f \). Several cost sharing mechanisms are possible but the simplest, and probably the most commonly used in practice, is proportional allocation. With this mechanism, the share of the cost \( c_r(x_r) \) of resource \( r \) which must be absorbed by an activity \( a \) is directly proportional to the quantity \( x_{ai} \) of resource \( r \) it uses, i.e., the unit cost charged for resource \( r \) is \( c_r(x_r)/x_r^* \), where \( x_r^* \) is the total
amount of resource $r$ actually used by the enterprise and $x_r^o = \sum_{a \in A_f^j} x_{ra}$. The cost sharing function under the assumption of proportional allocation can thus be written as follows:

$$C_f = \sum_{r \in R_f} \left( \frac{x_{ra}}{x_r^o} C_r(x_r) \right) - u_a = 0, \quad a \in A_f^i;$$

Hence, the last term $\sum_{a \in A_f^j} u_a$ of the objective function of GP reduces to

$$\sum_{a \in A_f^j} u_a = \sum_{a \in A_f^j} \sum_{r \in R_f} \left( \frac{x_{ra}}{x_r^o} C_r(x_r) \right) = \sum_{r \in R_f} \frac{C_r(x_r)}{x_r^o} \sum_{a \in A_f^j} x_{ra} = \sum_{r \in R_f} C_r(x_r).$$

This shows that, under the proportionality assumption, the computation of the value added by the enterprise is independent of the cost sharing mechanism used: it depends simply on the amount paid for the resources of the enterprise.

The metric and cost function appropriate for a given resource type can be quite diverse. Here, to keep things relatively simple, it is assumed that the resources used by the enterprise are divisible, and that the amount of resource $r$ available at the beginning of the horizon, $x_r^o$, costs a known amount, $c_r^o$, and can be adjusted within certain limits at a linear cost. The type of function considered is illustrated in Fig. 1. Under this assumption, the domains $\tilde{X}_r^+$ and $\tilde{X}_r^-$ reduce to the intervals

$$0 \leq x_r^+ \leq X_r^+ \quad \text{and} \quad 0 \leq x_r^- \leq X_r^-, \quad r \in R_f^1, \quad (4)$$

where $X_r^+$ is the maximal possible resource increase and $X_r^-$ is the maximal possible resource decrease; and the cost of resource $r$ is modeled by the linear function

$$c_r(x_r) = c_r^0 - \gamma_r^+ x_r^+ + \gamma_r^- x_r^-,$$

where $\gamma_r^+$ and $\gamma_r^-$ are the unit cost of a resource increase and decrease, respectively. It is assumed that, $\gamma_r^+ > \gamma_r^- \forall r$, to ensure that it does not pay to increase and decrease the amount of resources simultaneously.

These cost modeling assumptions are reasonable when the enterprise wants to make only marginal changes to its current resource base. When the enterprise is considering the acquisition of new types of resources (related to the implementation of a new technology or a new facility, for example), more complex cost functions, reflecting fixed costs and economies of scale, must be used. The approach remains the same, but the resulting model is more complex.

### 3.3. Product value functions

Porter (1985) defines value as “the amount buyers are willing to pay for what a firm provides them”. He adds: “value is measured by total revenue, a reflection of the price a firm’s product commands and the units it can sell”. The total revenue generated by the transactions discussed in this paper is generally the result of negotiations between the parties or of market forces. It is assumed that these processes determine the nature of the value functions associated to the external network arcs and that piecewise linear functions of the type illustrated in Fig. 2 are sufficiently general to capture most practical situations. In practice, negotiations between parties often result in pricing schemes including volume discounts and discontinuities, but piecewise linearity is sought to simplify implementation.

Using the modeling approach introduced by Kettani (1988), the value function associated to external arc $(i,j,p)$ can be written as

![Fig. 1. Resource cost function.](image-url)
v_{ijp}(y_{ijp}) = \sum_{k \in K_{ijp}} x_{ijp}^k \left(f_{ijp}^k + v_{ijp}^k y_{ijp}\right), \quad \sum_{k \in K_{ijp}} x_{ijp}^k = 1,  \\
(5)

where $K_{ijp}$ is the set of intervals on the $y$-axis of the value function, $f_{ijp}^k$ and $v_{ijp}^k$ are the value at the origin and the slope of the linear function in interval $k \in K_{ijp}$ and $x_{ijp}^k$ is the binary variable

$x_{ijp}^k = \begin{cases} 1 & \text{if } L_{ijp}^k \leq y_{ijp} < U_{ijp}^k \quad \forall k \in K_{ijp}, \\ 0 & \text{otherwise} \end{cases}$

(6)

where $L_{ijp}^k$ and $U_{ijp}^k$ are, respectively, the lower and the upper bounds of interval $k$ on the $y$-axis.

The term $x_{ijp}^k v_{ijp}^k y_{ijp}$ in (5) is quadratic but it can be linearized by replacing it with a non-negative continuous variable $\mathcal{L}_{ijp}^k$, as suggested by Oral and Kettani (1992). When the value function $v_{ijp}(y_{ijp})$ must be minimized, which is the case for external input arcs, then (5) and (6) can be replaced by the following relations:

$v_{ijp}(y_{ijp}) = \sum_{k \in K_{ijp}} \left(f_{ijp}^k x_{ijp}^k + \mathcal{L}_{ijp}^k\right), \quad \sum_{k \in K_{ijp}} x_{ijp}^k = 1,  \\
x_{ijp}^k \in \{0, 1\},  \\
(7)

\mathcal{L}_{ijp}^k \geq v_{ijp}^k y_{ijp} - v_{ijp}^k y_{ijp}^\text{max} (1 - x_{ijp}^k),  \\
\mathcal{L}_{ijp}^k \geq 0 \quad \forall k \in K_{ijp},  \\
(8)

where $y_{ijp}^\text{max}$ is the maximum value which can be taken by $y_{ijp}$, as previously defined. Note that since $y_{ijp}^\text{max}$ corresponds to the upper bound of the last interval, as shown in Fig. 2, the constraints (9) automatically ensure that $0 \leq y_{ijp} \leq y_{ijp}^\text{max}$ so that the bounds (2) are redundant.

When the value function must be maximized, which is the case for external output arcs, then (7) and (8) are replaced by

$v_{ijp}(y_{ijp}) = \sum_{k \in K_{ijp}} \left(f_{ijp}^k + v_{ijp}^k y_{ijp}^\text{max}\right) x_{ijp}^k - \mathcal{L}_{ijp}^k,  \\
(9)

\sum_{k \in K_{ijp}} x_{ijp}^k = 1, \quad x_{ijp}^k \in \{0, 1\},  \\
(10)

\mathcal{L}_{ijp}^k \geq v_{ijp}^k y_{ijp}^\text{max} x_{ijp}^k - v_{ijp}^k y_{ijp}, \quad \mathcal{L}_{ijp}^k \geq 0 \quad \forall k \in K_{ijp}.  \\
(11)

3.4. Networking model

Taking the previous discussion into account, it can be seen that the optimal networking strategy of a typical production-distribution company is obtained by solving the following model:

Fig. 2. Product value function.
max \( w_f \)
\[
= \sum_{(a,p) \in S_f^0} \sum_{k \in K_{a,p}} \left( f_{a,p}^k + s_{a,p}^{\max} \right) \gamma_{a,p}^k - \gamma_{a,p}^k\\
- \sum_{(a,p) \in S_f^0} \sum_{k \in K_{a,p}} \left( f_{a,p}^k \gamma_{a,p}^k + \gamma_{a,p}^k \right)\\
- \sum_{r \in R_f} \left( c_r^0 - x_r^0 - x_r^+ x_r^+ \right) \\
\text{(MIP)}
\]

subject to

- production and demand constraints (MIP-1)
\[
E_{a}^p y_i^a - B_a y_i^0 = 0 \quad \forall a \in A_f^1,\\
x_a - G_{a}^p y_i^a = 0 \quad \forall a \in A_f^1,
\]
\[
d_{pa} \leq \sum_{(i,a,p) \in S_f^0} y_{iap} \leq \bar{d}_{pa} \quad \forall a \in A_f^1, p \in P_f^0,\\
y_{ijp} \geq 0 \quad \forall (i,j,p) \in S_f,
\]

- value function definition constraints (MIP-2)
\[
\sum_{k \in K_{a,p}} L_{ijp}^k \gamma_{iap}^k \leq y_{ijp} \leq \sum_{k \in K_{a,p}} U_{ijp}^k \gamma_{iap}^k \\
\forall (i,j,p) \in S_f^0 \cup S_f^1,
\]
\[
\gamma_{iap}^k = 1 \quad \forall (i,j,p) \in S_f^0 \cup S_f^1,\\
L_{iap}^k \geq \gamma_{iap}^k v_{iap} - \gamma_{iap}^k v_{iap}^{\max} x_{iap}^k \\
\forall (a,j,p) \in S_f^0, k \in K_{a,p},\\
\gamma_{iap}^k \geq v_{iap} y_{iap} - v_{iap}^{\max} (1 - x_{iap}^k) \\
\forall (a,j,p) \in S_f^1, k \in K_{a,p},\\
\gamma_{iap}^k \in \{0,1\},\\
\gamma_{iap}^k \geq 0 \quad \forall (i,j,p) \in S_f^0 \cup S_f^1, k \in K_{a,p},
\]

- capacity constraints (MIP-3)
\[
\sum_{a \in A_f^1} x_a + x_r^+ - x_r^+ \leq v_{iap}^0 \quad \forall r \in R_f^1,
\]
\[
0 \leq x_r^+ \leq X_r^+ \quad \forall r \in R_f^1
\]

\[ x_{iap} \geq 0 \quad \forall a \in A_f^1, r \in R_f^1. \]

This is a large scale mixed integer programming (MIP) problem. Its objective function and its constraints are linear, but they involve a large number of 0–1 variables.

4. Solution method

Program MIP is difficult to solve optimally for realistic problems, which are usually very large. For this reason, a heuristic method is proposed which can be solved with a commercial solver such as CPLEX. The heuristic has two phases. In the first phase, the number of 0–1 variables is reduced by eliminating regions of the value functions which are likely not to be used in an optimal solution, because of demand and capacity constraints propagation effects. In the second phase, 0–1 variables are fixed by obtaining a series of improved feasible solutions and by inspecting them to determine if the flow of products on external arcs fall in the lowest cost, or highest profit, segment of the value functions associated to the product considered. This second phase is based on a well-known idea used by Kuzdrall and Britney (1982) to solve single-item lot-sizing problems with quantity discounts.

Let \( V_{iap}^* \) be the smallest unit cost which could be paid for the external supply of product \( p \) to activity \( a \), i.e., let
\[
V_{iap}^* = \max_{i,k} \left( V_{iap}^k > 0 \right), \quad (i,a,p) \in S_f
\]
and let \( V_{iap}^0 \) be the largest unit price which could be obtained on the market for product \( p \) coming from activity \( a \), i.e., let
\[
V_{iap}^0 = \max_{i,k} \left( V_{iap}^k \right), \quad (a,j,p) \in S_f^0
\]
Also, let \( S_f^1 \subset S_f \) be the set of the arcs on which the best value segments are found, \( k_{iap} \) be the index of the selected segment for arc \( (i,j,p) \in S_f^1 \) and \( L_{iap}^k \) and \( U_{iap}^k \) be the lower and upper bounds of segment \( k_{iap} \), respectively.

In the first phase of the heuristic, in order to determine the sub-domain of interest for each
value function, a linear program obtained by replacing the value function of every external arc by the best possible value \( V^*_a \) or \( V_0 \) of its product is solved, i.e.,

\[
\max \sum_{(i,j) \in S_f^p} v_{ijp} - \sum_{(i,a) \in S_f^a} v_{iap} y_{iap} - \sum_{r \in R_f} \left( c^0_r - y^*_r x^*_r + y^+_r x^+_r \right) \quad \text{(LP)}
\]

subject to

- the production/demand constraints MIP-1,
- the supply constraints
  \[
y_{iap} \leq y_{iap}^\text{max} \quad \forall (i,j,p) \in S_f^p,
\]
- the capacity constraints MIP-3.

Let \( y_{iap}^\text{max} \) denote the optimal flows obtained by solving LP. The rational behind the first phase is the following: the maximum quantity of product \( p \) which should be required by an activity \( a \) at the end of a sourcing arc is \( \sum_{i \in d_i} y_{iap}^* \); hence, any segments of the value function on the arcs \( (i,a,p) \in S_a^p \) which applies to flow quantities higher than this total should not be relevant. Consequently, for all the sourcing arcs, the value functions are redefined by removing any irrelevant intervals from the sets \( K_{iap} \), and by setting

\[
y_{iap}^\text{max} = U_{iap}^{|K_{iap}|} = \min(y_{iap}^\text{max}, \sum_{i \in d_i} y_{iap}^*), \quad (i,a,p) \in S_f^p.
\]

The MIP obtained by revising the value functions in this way is denoted MIP_\( \hat{s} \).

The second phase is iterative and it involves the partial solution of MIP_\( s \), a version of MIP_\( \hat{s} \) in which some of the 0–1 variables have been fixed. At the first iteration, the program to solve is MIP_\( \hat{s} \). To keep running time low, the solution procedure is stopped as soon as a better feasible solution is found, or the computation time reaches a predetermined maximum MCT_\( s \). Let \( y_{iap}^s \) denote the flows of the new feasible solution obtained at iteration \( s \). If for some arcs \( (i,a,p) \in S_f^p \), the flow obtained falls in the best value interval, i.e., if \( \left( U_{iap}^{|K_{iap}|} \leq y_{iap}^s \leq U_{iap}^{|K_{iap}|} \right) \), then \( y_{iap}^s = 1 \) and \( y_{iap}^k = 0 \) \( \forall k \neq k_{iap} \) are set. This defines MIP_{\( s+1 \)}, the next program to solve. The procedure continues like this until an optimal solution for the current MIP is found.

The complete algorithm is summarized in Fig. 3. Note that when a product can come from an internal as well as an external source, it is not possible to determine the lowest cost because the value function for internal products is not available. This is why in step (5) of the heuristic, only arcs associated with a product \( p \) which cannot come from an internal activity \( (p \notin P^i \cap P^f) \) are considered. It is also possible that the feasible solution of MIP_\( s \) does not have any flow variables which fall in a best value interval not already fixed. When this is the case, MIP_{\( s+1 \)} is identical to MIP_\( s \). One approach to avoid this would be to consider fixing the 0–1 variables when a flow variable falls into the second best value interval.

5. Applicability and potential for strategic planning

The objective of this section is to show that the networking model can be solved efficiently, for realistic problems, and to illustrate how the model can be used for strategic planning. A fictitious extended enterprise based on the supply chain of a refrigerator manufacturing company is used as an illustration. A detailed description of the enterprise activities, resources and costs is given in Lakhal et al. (1998). The supply chain of the enterprise includes 15 internal primary activities, one internal support activity and 24 potential external activities. Opportunities exist to externalize shelf coating, plastic injection and transportation activities. Several alternative suppliers are also available for raw materials. The company would like to investigate the impact of certain marketing strategies on its networking decisions. Also, a new labor agreement needs to be negotiated and the company would like to investigate the interest of the sub-contracting opportunities available under various labor conditions.

In order to solve MIP for this case, a Model Generator and Report Writer was developed in the Delphi Environment on a PC. The application was programmed in Pascal and a DLL was written to link it to the CPLEX 4.1 solver. For the various scenarios studied, the initial MIP had 110 binary
variables, 270 continuous variables and 380 constraints. A few hours were required to find an optimal solution using CPLEX 4.1 directly. With the heuristic proposed, however, near-optimal solutions were found in about 100 seconds. When the value functions did not involve any fixed costs, the heuristic always gave the optimal solution. Otherwise, the solutions found were within 3% of the optimum. In all cases, the parameter MCT2 was given a value large enough to ensure that the algorithm would stop after finding the optimal solution for MIP2.

To illustrate the potential of the approach for strategy formulation, several potential scenarios were elaborated, each with a particular issue in mind. The optimal value of the solutions obtained are reported in Table 1 for the five main scenarios studied, and each is briefly discussed in what follows. The first issue considered is the impact of the marketing strategy adopted on the network structure. The three first scenarios are related to this issue.

The first Scenario assumes that the firm does not impose any minimal market penetration targets \( \hat{d}_{pa} \) and that the demand is unbounded \( \hat{d}_{pa} = \infty \). In other words, given the refrigerator pricing policy of the company and the resource redeployment and outsourcing opportunities
available, what networking strategy should be adopted? In such a context, the strategy which maximizes profits is to produce as much as possible, and using external partners to supplement the internal capacity available: a plastic injection subcontractor is used in addition to the internal injection activity to increase capacity by 67% and public transporters are used for both internal and outbound transportation. No activity is completely externalized.

Now, in contrast, Scenario 2 assumes that 20,000 refrigerators can be sold during the planning horizon, with the product-market breakdown provided in Table 1. Under this assumption, revenues earned are a constant and solving MIP requires minimizing total cost of input products and resources required to meet the forecast demand. Naturally, optimal profits generated under this scenario are much lower than for Scenario 1 ($1.3 millions instead of $1.86). Shelve coating activity should be performed internally, but the production of vegetable trays should be totally outsourced. Internal and outbound transportation should be partially externalized.

In most cases, companies would rather adopt a marketing strategy which falls in-between these two approaches: for each product-market, a minimum penetration target is fixed, and sales are limited by an upper bound based on total market potential and on historical market shares. The company then has to decide how much of each finished product it should make in order to maximize profits. This is the approach assumed in Scenario 3. The solution obtained under this scenario suggests producing 24,762 refrigerators, as for Scenario 1, but the profit made under the current marketing policy is slightly lower. The main difference between the two solutions is that the shelves coating activity must now be partially externalized. A first conclusion, which stems from the study of these three scenarios, is that the best networking strategy for the company depends on product-market potential and on marketing strategy. Different marketing contexts lead to different networking and resource deployment decisions, and value added cannot be maximized if this is not exploited.

Several other issues can be explored with the model. Starting with the assumptions of Scenario 2, Scenario 4 assumes that labor costs are likely to increase from $25 to $28 per hour and the cost of additional personnel from $32.50 to $36.40 per hour. What could be the consequences of such an event? As shown by the results in Table 1, an obvious consequence is that, all other things being equal, profit would decrease from $1.298 to $0.895 millions. In this case, the company is better off if it completely outsources the shelves plastication activity as well as the transportation of refrigerators from the warehouse to regional depots.

This gives rise to the next natural question. What would happen if internal resources were completely disposable without any penalty? In Scenario 5, the bounds on the subtraction of resources are assumed equal to current resource levels \((X^c_r = x^c_r \ \forall r \in R_f)\), and amounts recovered equal to current resource unit costs \((\gamma^c_r = c^c_r / x^c_r \ \forall r \in R_f)\). As expected, the first consequence of this additional flexibility is a significant increase in profits (from $1.3 to $1.45 millions). The interesting aspect of this scenario is that it shows which internal activities are not productive when compared to concurrent external activities. For instance, outbound transportation activity is not

<table>
<thead>
<tr>
<th>Market demand</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
<th>Scenario 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>((d^{26,31}, \bar{d}^{26,31}))</td>
<td>((0, \infty))</td>
<td>((6000, 6000))</td>
<td>((3000, 10000))</td>
<td>((6000, 6000))</td>
<td>((6000, 6000))</td>
</tr>
<tr>
<td>((d^{27,31}, \bar{d}^{27,31}))</td>
<td>((0, \infty))</td>
<td>((4000, 4000))</td>
<td>((2000, 8000))</td>
<td>((4000, 4000))</td>
<td>((4000, 4000))</td>
</tr>
<tr>
<td>((d^{26,34}, \bar{d}^{26,34}))</td>
<td>((0, \infty))</td>
<td>((8000, 8000))</td>
<td>((4000, 12,000))</td>
<td>((8000, 8000))</td>
<td>((8000, 8000))</td>
</tr>
<tr>
<td>((d^{27,34}, \bar{d}^{27,34}))</td>
<td>((0, \infty))</td>
<td>((2000, 2000))</td>
<td>((1000, 5000))</td>
<td>((2000, 2000))</td>
<td>((2000, 2000))</td>
</tr>
<tr>
<td>Total production</td>
<td>24,762</td>
<td>20,000</td>
<td>24,762</td>
<td>20,000</td>
<td>20,000</td>
</tr>
<tr>
<td>Profit ($)</td>
<td>1,864,571</td>
<td>1,298,844</td>
<td>1,797,989</td>
<td>895,107</td>
<td>1,447,693</td>
</tr>
</tbody>
</table>
competitive, which suggests that the firm should either externalize it, or take steps to improve transportation methods. It also highlights competitive internal activities. In this case, for example, the production of butter compartment doors is entirely internalized, which indicates that the plastic injection methods used to produce doors are adequate.

6. Concluding remarks

As demonstrated in the previous section, the model proposed in this paper can effectively support the elaboration of the networking and resource deployment strategy of an enterprise. The model is a large MIP problem, but a heuristic which gives good quality solutions in a reasonable amount of time was designed to make the tool practical.

One important limitation of our model is that it is static. The dynamics of supply chains need to be taken into consideration while investigating the type of strategic issues raised in the previous section. Supply chains are living organisms: the resources used and the activities performed by companies change in time as a result of the decisions made to compete better. This also raises the necessity of explicitly considering competing companies, as well as the impact of learning within the network company. These are suggested issues for future research.

Acknowledgements

This research was supported by the Social Sciences and Humanities Research Council of Canada.

References


